

# Industrial Applications of PFAST

Product-Process Matrix Analysis

FAST

# Product–Process Matrix Analysis (PR-I Analysis)

---

## Description

The reorganization of an existing layout into a Cellular Layout consisting of independent manufacturing cells is an important strategy for reducing WIP and order delivery times in a multi-machine multi-product discrete manufacturing facility. This tool helps to identify machine<sup>1</sup> groups and product<sup>2</sup> families which serve as a foundation for design of manufacturing cells.

## Typical Application

- To form the group of machines that will constitute a cell
- To form the product family that will be processed by this cell

## Overview of Methodology

- Collect data:
  - Product routings
  - Plant List (List of machines in the facility)
- Form the initial 0-1 Product-Process matrix<sup>3</sup>
- Derive the final 0-1 Product-Process matrix using the PR Analysis Type I results in the PFAST Analysis Report
- Identify the Block Diagonal Form<sup>4</sup> (BDF)
- Use the BDF to form machine groups and product families

## Step-by-Step Procedure for Implementation

STEP 1: Select the sample of products from the entire population of products using PQ Analysis, P\$ Analysis and/or PQ\$ Analysis

STEP 2: Obtain the routings of these products

STEP 3: Use the product routings to form the initial 0-1 Product-Process matrix

STEP 4: Derive the final 0-1 Product-Process matrix using the PR Analysis Type 1 results in the PFAST Analysis Report

---

<sup>1</sup> *Machine, Process, Workcenter* – These terms can be used interchangeably.

<sup>2</sup> *Product, Part, Component* – These terms can be used interchangeably.

<sup>3</sup> *0 -1 Product-Process Matrix*: A Product-Process matrix ( $a_{ij}$ ) consists of 0's and 1's depending on whether or not a component is processed on a machine. Hence,  $a_{ij}$  is 1 if component  $j$  is processed on machine  $i$ , and 0, if otherwise.

<sup>4</sup> *Block Diagonal Form (BDF)*: BDF is the final matrix that results after rearranging the initial 0-1 Product-Process matrix so as to pack the 1's in the matrix into dense blocks of 1's along the diagonal of the matrix. Each block in the BDF matches a product family with a group of machines that could be co-located into a manufacturing cell dedicated to the manufacture of the products in that family.

STEP 5: Identify the machine cells and product families by partitioning the final Product-Process matrix into a BDF

### Illustrative Example

**Steps 1 and 2:** A sample of 19 products from the entire population of products is selected using PQ Analysis, P\$ Analysis and/or PQ\$ Analysis. The routings of these products are shown in Figure 1.

Part No.	Qty	Op 1	Op 2	Op 3	Op 4	Op 5	Op 6	Op 7
Part 1	10642	1	4	8	9			
Part 2	4270	1	4	7	4	8	7	
Part 3	1471	1	2	4	7	8	9	
Part 4	4364	1	4	7	9			
Part 5	5013	1	6	10	7	9		
Part 6	4679	6	10	7	8	9		
Part 7	5448	6	4	8	9			
Part 8	5339	3	5	2	6	4	8	9
Part 9	9117	3	5	6	4	8	9	
Part 10	8935	4	7	4	8			
Part 11	7100	6						
Part 12	8611	11	7	12				
Part 13	9933	11	12					
Part 14	3824	11	7	10				
Part 15	1359	1	7	11	10	11	12	
Part 16	1235	1	7	11	10	11	12	
Part 17	8581	11	7	12				
Part 18	3963	6	7	10				
Part 19	2309	12						

**Figure 1 Routings of the Products**

**Step 3:** These product routings are used by PFAST to generate the initial 0-1 Product-Process matrix shown in Figure 2.

**Step 4:** The final Product-Process matrix, shown in Figure 3a, is derived from the PR Analysis Type I results in the PFAST Analysis Report.

**Step 5:** The machine cells and product families can be formed by inspecting the Block Diagonal Form (BDF) of the matrix shown in Figure 3b.

	m1	m2	m3	m4	m5	m6	m7	m8	m9	m10	m11	m12
Part 1	1			1				1	1			
Part 2	1			1			1	1				
Part 3	1	1		1			1	1	1			
Part 4	1			1			1		1			
Part 5	1					1	1		1	1		
Part 6						1	1	1	1	1		
Part 7				1		1		1	1			
Part 8		1	1	1	1	1		1	1			
Part 9			1	1	1	1		1	1			
Part 10				1			1	1				
Part 11						1						
Part 12							1				1	1
Part 13											1	1
Part 14							1			1	1	
Part 15	1						1			1	1	1
Part 16	1						1			1	1	1
Part 17							1				1	1
Part 18						1	1			1		
Part 19												1

**Figure 2 Initial Product-Process Matrix**

	m2	m3	m5	m4	m8	m9	m6	m1	m7	m10	m11	m12
Part 8	1	1	1	1	1	1	1					
Part 9		1	1	1	1	1	1					
Part 7				1	1	1	1					
Part 1				1	1	1		1				
Part 3	1			1	1	1		1	1			
Part 4				1		1		1	1			
Part 2				1	1			1	1			
Part 10				1	1				1			
Part 5						1	1	1	1	1		
Part 6					1	1	1		1	1		
Part 18							1		1	1		
Part 11							1					
Part 14									1	1	1	
Part 15								1	1	1	1	1
Part 16								1	1	1	1	1
Part 12									1		1	1
Part 17									1		1	1
Part 13											1	1
Part 19												1

**Figure 3a Final Product-Process Matrix**

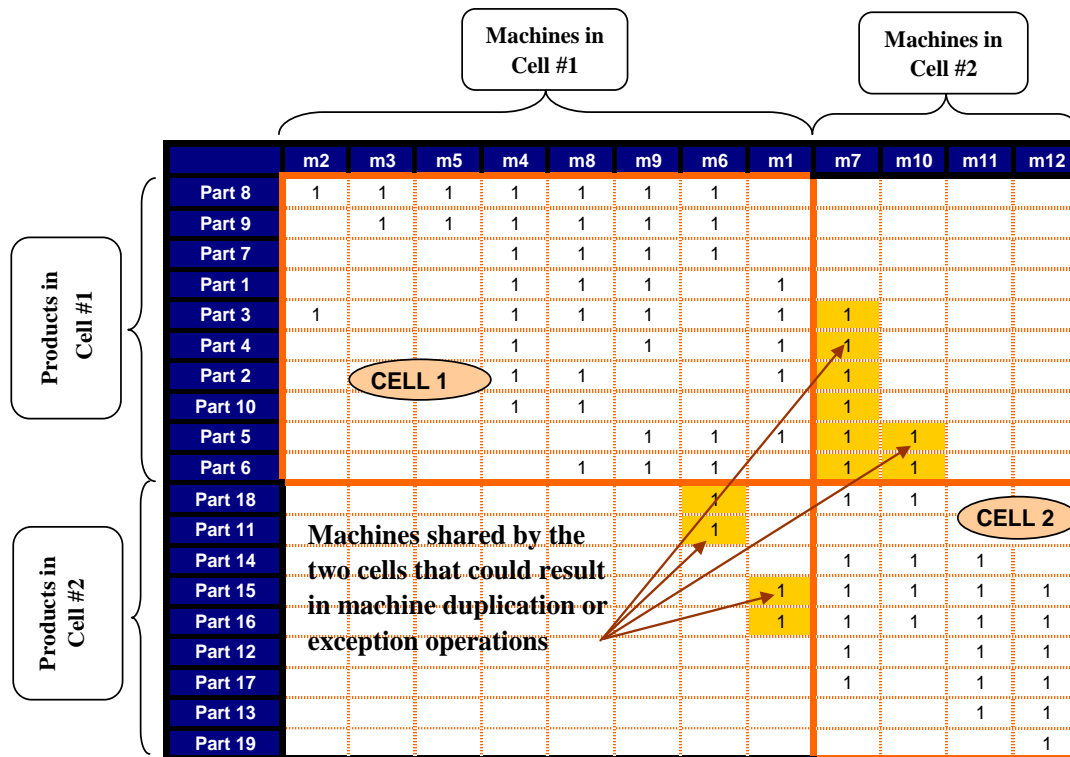


Figure 3b Block Diagonal Form (BDF) for Final Product-Process Matrix

## Using the Product – Process Matrix Analysis to Design a Cellular Layout

### Use 1: To Identify the Machine Group and Product Family for each Cell

The machine group and product family corresponding to each cell can be identified using the Block Diagonal Form (BDF) of the final Product-Process matrix. From Figure 4, Cell 1 has machines m2, m3, m5, m4, m8, m9, m6, and m1 to process products 8, 9, 7, 1, 3, 4, 2, 10, 5 and 6. Similarly, Cell 2 has machines m7, m10, m11 and m12 to process products 18, 11, 14, 15, 16, 12, 17, 13 and 19.

		Machines in Cell #1							Machines in Cell #2				
		m2	m3	m5	m4	m8	m9	m6	m1	m7	m10	m11	m12
Products in Cell #1	Part 8	1	1	1	1	1	1	1	1				
	Part 9		1	1	1	1	1	1	1				
	Part 7				1	1	1	1					
	Part 1				1	1	1		1				
	Part 3	1			1	1	1		1	1			
	Part 4				1		1		1	1			
	Part 2				1	1			1	1			
	Part 10				1	1				1			
	Part 5						1	1	1	1	1		
	Part 6					1	1	1		1	1		
Products in Cell #2	Part 18							1		1	1		
	Part 11							1					
	Part 14								1	1	1		
	Part 15							1	1	1	1	1	
	Part 16							1	1	1	1	1	
	Part 12								1		1	1	
	Part 17								1		1	1	
	Part 13										1	1	
	Part 19											1	1

Figure 4 Identification of Machine Groups and Product Families

### Use 2: To Identify the Exception Operations

Exception operations are those operations processed on a machine that does not belong in a cell. From Figure 5, Product 18 belongs to Cell 2. However, Machine 6, which is required for processing Product 18, is located in Cell 1. Hence, this operation is an exception operation. Similarly, other exception operations can be identified using the final Product-Process matrix.

	m2	m3	m5	m4	m8	m9	m6	m1	m7	m10	m11	m12
Part 8	1	1	1	1	1	1	1					
Part 9		1	1	1	1	1	1					
Part 7				1	1	1	1					
Part 1				1	1	1		1				
Part 3	1			1	1	1		1	1			
Part 4				1		1		1	1			
Part 2				1	1			1	1			
Part 10				1	1				1			
Part 5						1	1	1	1	1		
Part 6					1	1	1	1	1	1		
Part 18							1		1	1		
Part 11							1					
Part 14									1	1	1	
Part 15								1	1	1	1	1
Part 16								1	1	1	1	1
Part 12									1		1	1
Part 17									1		1	1
Part 13											1	1
Part 19												1

Figure 5 Identification of Exception Operations

Use 3: To Identify the Shared Machines

Shared<sup>5</sup> machines are those required by more than one cell. With the exception operations already identified as shown in Figure 5, and information on the available number of machines of each type, the shared machines can be identified. From Figure 6, if only one copy of Machine m7 is available, then the available capacity for m7 will have to be shared between the two cells.

	m2	m3	m5	m4	m8	m9	m6	m1	m7	m10	m11	m12
Part 8	1	1	1	1	1	1	1					
Part 9		1	1	1	1	1	1					
Part 7				1	1	1	1					
Part 1				1	1	1		1				
Part 3	1			1	1	1		1	1			
Part 4				1		1		1	1			
Part 2				1	1			1	1			
Part 10				1	1				1			
Part 5						1	1	1	1	1		
Part 6					1	1	1	1	1	1		
Part 18							1		1	1		
Part 11							1					
Part 14									1	1	1	
Part 15								1	1	1	1	1
Part 16								1	1	1	1	1
Part 12									1		1	1
Part 17									1		1	1
Part 13											1	1
Part 19												1

Figure 6 Identification of Shared Machines

<sup>5</sup> A shared machine is a machine that processes a significant number of products from several cells. An exception machine, on the other hand, processes a majority of products belonging to one cell and relatively few products from other cells. In many cases, an exception operation can be eliminated by rerouting it to another machine located within the cell.

#### Use 4: To Determine the Number of Cells

The final Product-Process matrix can be used to identify the number of cells by inspecting the structure of the matrix. For example, the 3-cell solution shown in Figure 7 can be initially obtained from the final Product-Process matrix that was derived using the PR Analysis Type I results in the PFAST Analysis Report.

	m3	m5	m2	m4	m8	m9	m6	m1	m7	m10	m11	m12
Part 11							1					
Part 8	1	1	1	1	1	1	1					
Part 9	1	1		1	1	1	1					
Part 7				1	1	1	1					
Part 1				1	1	1		1				
Part 3			1	1	1	1		1				
Part 10				1	1				1			
Part 2				1	1			1	1			
Part 4				1		1		1	1			
Part 6					1	1	1	1	1	1		
Part 5						1	1	1	1	1		
Part 18							1		1	1		
Part 14									1	1	1	
Part 16								1	1	1	1	1
Part 15								1	1	1	1	1
Part 17									1		1	1
Part 12									1		1	1
Part 13											1	1
Part 19												

Figure 7 Identification of the Number of Cells

#### Use 5: To Evaluate Alternative Cell Configurations

Different cell configurations can be evaluated using the final Product-Process matrix. Figure 8 shows an alternative 2-cell layout for the same facility.

	m2	m3	m5	m4	m8	m9	m6	m1	m7	m10	m11	m12
Part 8	1	1	1	1	1	1	1					
Part 9		1	1	1	1	1	1					
Part 7				1	1	1	1					
Part 1				1	1	1		1				
Part 3	1			1	1	1		1	1			
Part 4				1		1		1	1			
Part 2				1	1			1	1			
Part 10				1	1				1			
Part 5						1	1	1	1	1		
Part 6					1	1	1		1	1		
Part 18							1		1	1		
Part 11							1					
Part 14									1	1	1	
Part 15								1	1	1	1	1
Part 16								1	1	1	1	1
Part 12									1		1	1
Part 17									1		1	1
Part 13											1	1
Part 19												1

Figure 8 Evaluation of Alternative Cell Configurations



In the new configuration, machine m7 which was previously in Cell 2, has now been included in Cell 1. Using a spreadsheet software or performance evaluation software, such as MPX ([www.networkdyn.com/MPXWindows.html](http://www.networkdyn.com/MPXWindows.html)), ARENA ([www.software.rockwell.com/arenasimulation/](http://www.software.rockwell.com/arenasimulation/)) or PROMODEL ([www.promodel.com](http://www.promodel.com)) to perform static/dynamic capacity calculations, the pros and cons of different cell configurations can be compared to choose the best cell configuration.

#### **Use 6: To Facilitate Classification of Products**

The final Product-Process matrix can be used to classify products that (1) use only unshared machines, (2) use only shared machines, and (3) use both shared and unshared machines. From Figure 6, products 8, 9, 7, 1, 13 and 19 use unshared machines only; product 11 uses only shared machines; and the remaining products use both shared and unshared machines. This classification helps to identify the critical products that could prevent the design and operation of independent cells viz. the ones that require only shared machines and the ones that require both shared and unshared machines.

#### **Use 7: To Identify Inter-related Cells**

Inter-related cells are those that use one (or more) shared machine(s) or machines that are located in a Common Resource Cell. Shared machines are those machines that process products belonging to more than one cell. A Common Resource Cell is a cell that contains machines that are required by more than one cell. A shared machine can also be in a Common Resource Cell. The information on machine cells, shared machines and machines in the Common Resource Cell obtained from the Product-Process matrix can be used to design the overall layout for the inter-related cells (inter-cell layout).

#### **Use 8: To Provide Input to Subsequent Planning Decisions**

The final Product-Process matrix provides valuable input for subsequent decisions relating to detailed cell design, such as:

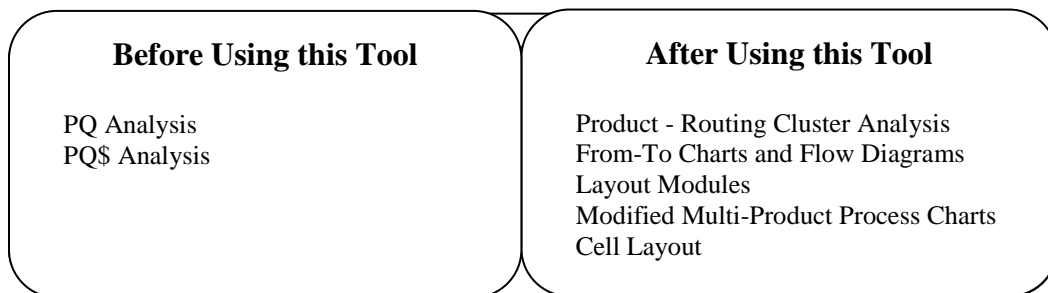
- Load balancing by machine duplication and allocation of parts
- Purchase of new machines
- Intra-cell layout, handling and work scheduling
- Inter-cell layout, handling and work scheduling

However, additional data such as setup times, processing times, available number of machines of each type, etc. must be obtained to do these detailed analyses for comprehensive design and performance evaluation of a manufacturing cell. The current version of PFAST lacks these capabilities. External programs perform the necessary calculations for Capacity Requirements Planning, Machine Requirements Analysis, Intra-cell Layout and Inter-cell Layout.

## Future Enhancements in this Tool

1. The number of cells must be known beforehand in order to determine the machine groups and product families. The tool must be enhanced to determine the optimal number of cells.
2. The partitioning of the BDF needs to be automated to enable this tool to be applied to large data sets.
3. The tool should be enhanced to handle data such as product routings, production volumes and machine capacities.

## Links to other Tools



## References

Irani, S.A. (Editor). (1999). Handbook of Cellular Manufacturing Systems. New York, NY: John Wiley.

Irani, S. A. (Autumn 2004). Course Packet for ISE 532 – Production Systems II, Department of Industrial and Systems Engineering, The Ohio State University, Columbus, OH.

Irani, S.A. and Huang, H. (August 2004). Hybrid Cellular Layouts – New Ideas for Design of Flexible and Lean Layouts for Jobshops. <http://www.zipedu.com/HCL.html>.

## Websites

PFAST: <http://cast.cse.ohio-state.edu/pfast>

Future Manufacturing and Production Facilities: <http://www-iwse.eng.ohio-state.edu/ISEFACULTY/IRANI/IRANI.htm>

## APPENDIX

### Strategies to Eliminate or Reduce the Inter-cell Flows in a Cellular Layout

Sl. No	Strategies to Eliminate/Reduce Inter-cell Flows	Figure	Illustrative Example
1	Redesign the products to eliminate the exception operations in their routings	A1	If parts 15 and 16 are redesigned in such a way that machine m1 is not required, then machine m1 can be placed exclusively in Cell 1
2	Combine cells that share machines	A2	If the two cells are merged to form a single cell, this would eliminate inter-cell flows for parts 2, 3, 4, 5, 6, 10, 15 and 16
3	Subcontract the exception operations or the complete part that has exception operations in its routing	A3	If the operations on machine m1 for parts 15 and 16 are subcontracted, the associated inter-cell flows can be eliminated
4	Buy extra machines and duplicate them among several cells that need them	A4	If machine m7 is duplicated in Cell 1, then the associated inter-cell flows due to machine m7 will be eliminated
5	Purchase multi-purpose machines (MPMs) to replace groups of machines in one or more cells	A5	In Cell 1, an MPM replaces the group of machines m3, m7 and m10, then the inter-cell flows due to parts 2, 3, 4, 5, 6 and 10 will be eliminated
6	Place shared machines in a Common Resource Cell (or Common Facility Cell) located centrally among the other cells	A6	If machines m1, m6, m7 and m10 are placed in a Common Resource Cell that is in close proximity to Cell 1 and Cell 2, then the inter-cell flow distances will be reduced
7	Seek alternative machine/s within the host cells that can do the exception operations	A7	Machine m11 in Cell 2 is capable of substituting for machine m6. This eliminates inter-cell flows for parts 11 and 18
8	Plan the shop layout to keep the cells close to each other	-	-
9	Utilize priority scheduling rules to signal the material handlers to move these parts with exception operations in their routings between cells on a JIT basis	-	-
10	Utilize priority scheduling rules that favor those parts requiring inter-cell moves in a cell's production schedule	-	-

	m2	m3	m5	m4	m8	m9	m6	m1	m7	m10	m11	m12
Part 8	1	1	1	1	1	1	1					
Part 9		1	1	1	1	1	1					
Part 7				1	1	1	1					
Part 1				1	1	1		1				
Part 3	1			1	1	1		1	1			
Part 4				1		1		1	1			
Part 2				1	1			1	1			
Part 10	<b>CELL 1</b>			1	1				1			
Part 5						1	1	1	1	1		
Part 6					1	1	1		1	1		
Part 18							1		1	1		
Part 11							1				<b>CELL 2</b>	
Part 14									1	1	1	
Part 15								0	1	1	1	1
Part 16								0	1	1	1	1
Part 12									1		1	1
Part 17									1		1	1
Part 13											1	1
Part 19												1

Figure A1 Parts 15 & 16 are Redesigned to Eliminate Operations on Machine m1

	m2	m3	m5	m4	m8	m9	m6	m1	m7	m10	m11	m12
Part 8	1	1	1	1	1	1	1					
Part 9		1	1	1	1	1	1					
Part 7				1	1	1	1					
Part 1				1	1	1		1				
Part 3	1			1	1	1		1	1			
Part 4				1		1		1	1			
Part 2				1	1			1	1			
Part 10	<b>CELL 1</b>			1	1			1	1			
Part 5						1	1	1	1	1		
Part 6					1	1	1	1	1	1		
Part 18							1	1	1	1		
Part 11							1	1				
Part 14								1	1	1		
Part 15								1	1	1	1	1
Part 16								1	1	1	1	1
Part 12									1		1	1
Part 17									1		1	1
Part 13											1	1
Part 19												1

Figure A2 Cell 1 and Cell 2 are Combined

	m2	m3	m5	m4	m8	m9	m6	m1	m7	m10	m11	m12
Part 8	1	1	1	1	1	1	1					
Part 9		1	1	1	1	1	1					
Part 7				1	1	1	1					
Part 1				1	1	1		1				
Part 3	1			1	1	1		1	1			
Part 4				1		1		1	1			
Part 2				1	1			1	1			
Part 10				1	1			1	1			
Part 5						1	1	1	1	1		
Part 6					1	1	1	1	1	1		
Part 18							1		1	1		
Part 11							1					
Part 14								1	1	1		
Part 15								1	1	1	1	1
Part 16								1	1	1	1	1
Part 12									1		1	1
Part 17									1		1	1
Part 13											1	1
Part 19												1

CELL 1

Exception operations for Parts 15 & 16 on Machine m1 can be subcontracted

CELL 2

Alternatively, Parts 15 & 16 can be completely subcontracted

Figure A3 Operations on Machine m1 for Parts 15 & 16 are Subcontracted

	m2	m3	m5	m4	m8	m9	m6	m1	m7(1)	m7(2)	m10	m11	m12
Part 8	1	1	1	1	1	1	1						
Part 9		1	1	1	1	1	1						
Part 7				1	1	1	1						
Part 1				1	1	1		1					
Part 3	1			1	1	1		1	1				
Part 4				1		1		1	1				
Part 2				1	1			1	1				
Part 10				1	1			1	1				
Part 5						1	1	1	1		1		
Part 6					1	1	1	1	1		1		
Part 18							1		1	1			
Part 11							1						
Part 14								0	1	1	1		
Part 15								0	1	1	1	1	
Part 16								0	1	1	1	1	
Part 12									1		1	1	
Part 17									1		1	1	
Part 13											1	1	
Part 19													1

CELL 1

Machine m7 is duplicated

These are not inter-cell moves anymore

CELL 2

Figure A4 Machine m7 is Duplicated in Cell 1

	m2	m3	m5	m4	m8	m9	m6	m3	m7	m10	m3	m7	m10	m11	m12
Part 8	1	1	1	1	1	1	1								
Part 9		1	1	1	1	1	1								
Part 7				1	1	1	1								
Part 1				1	1	1		1							
Part 3	1			1	1	1		1	1						
Part 4				1		1		1	1						
Part 2				1	1			1	1						
Part 10				1	1				1						
Part 5						1	1	1	1	1					
Part 6					1	1	1		1	1					
Part 18							1				1		1		
Part 11							1								
Part 14											1		1		
Part 15											1	1	1	1	1
Part 16											1	1	1	1	1
Part 12											1			1	1
Part 17											1			1	1
Part 13														1	1
Part 19															1

CELL 1 (Parts 1-10)

MPM for Cell 1 (Parts 1, 3, 4, 2, 10)

CELL 2 (Parts 11-19)

Figure A5 A Multi-Purpose Machine that Eliminates m3, m7 and m10 is Placed in Cell 1

	m2	m3	m5	m4	m8	m9	m6	m1	m7	m10	m11	m12
Part 8	1	1	1	1	1	1	1					
Part 9		1	1	1	1	1	1					
Part 7				1	1	1	1					
Part 1				1	1	1		1				
Part 3	1			1	1	1		1	1			
Part 4				1		1		1	1			
Part 2				1	1			1	1			
Part 10				1	1				1			
Part 5						1		1	1	1		
Part 6					1	1		1		1		
Part 18							1		1	1		
Part 11							1					
Part 14									1	1	1	
Part 15								1	1	1	1	1
Part 16								1	1	1	1	1
Part 12									1		1	1
Part 17									1		1	1
Part 13											1	1
Part 19												1

CELL 1 (Parts 1-10)

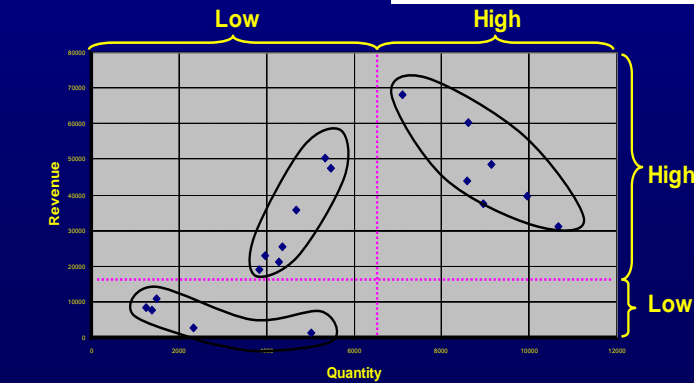
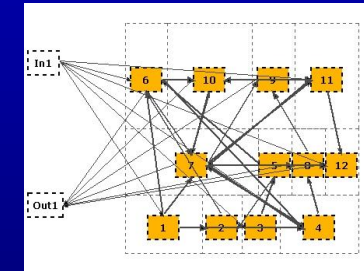
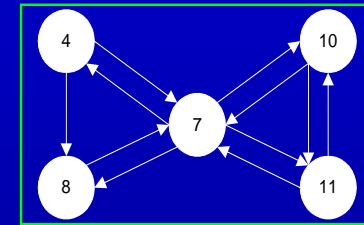
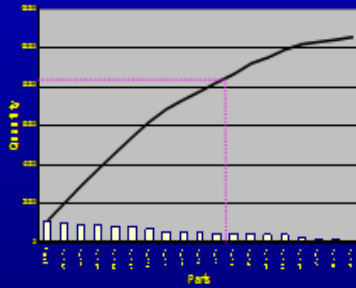
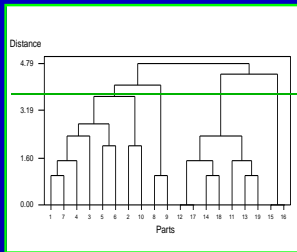
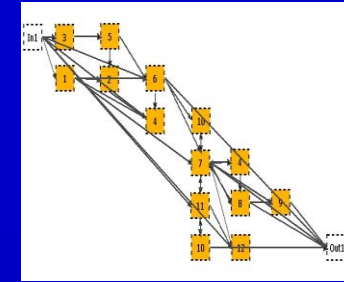
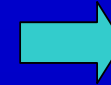
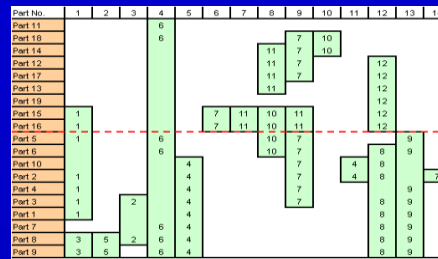
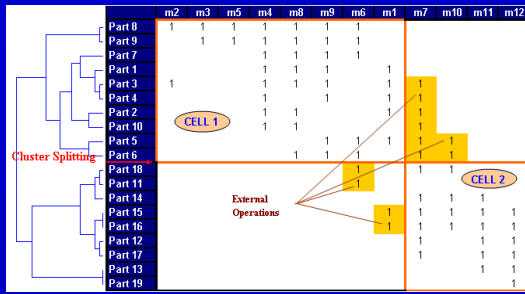
Common Resource Cell (Parts 11-19)

CELL 2 (Parts 11-19)

Figure A6 A Common Resource Cell is Established for Exception Operations

	m2	m3	m5	m4	m8	m9	m6	m1	m7	m10	m11	m12
									m6 m11			
Part 8	1	1	1	1	1	1	1					
Part 9		1	1	1	1	1	1					
Part 7				1	1	1	1					
Part 1				1	1	1		1				
Part 3	1			1	1	1		1	1			
Part 4				1		1		1	1			
Part 2				1	1			1	1			
Part 10	CELL 1			1	1				1			
Part 5						1	1	1	1	1		
Part 6					1	1	1		1	1		
Part 18									1	1	1	
Part 11											1	
Part 14									1	1		1
Part 15								1	1	1		1
Part 16								1	1	1		1
Part 12									1			1
Part 17									1		CELL 2	
Part 13											1	1
Part 19												1

Figure A7 Machine m11 is used as an Alternative to Machine m6 in Cell 2



Module	Common Substrings of Operations	
1	1 -> 4 -> 7	M1
1	4 -> 7 -> 4 -> 8	
1	6 -> 4 -> 8 -> 9	
1	7 -> 8 -> 9	M2
1	7 -> 9	
2	6 -> 10 -> 7	M3
2	7 -> 10	
2	1 -> 7 -> 11 -> 10 -> 11 -> 12	
2	11 -> 7 -> 12	
3	3 -> 5	

